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2005 J. Phys.: Condens. Matter 17 3895

(http://iopscience.iop.org/0953-8984/17/25/016)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **17** (2005) 3895–3901 doi:10.1088/0953-8984/17/25/016

Spin-wave softening and Hund's coupling in ferromagnetic manganites

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Received 1 April 2005, in final form 24 May 2005
Published 10 June 2005
Online at stacks.iop.org/JPhysCM/17/3895

Abstract

Using a one-orbital model of hole-doped manganites, we show with the help of the Holstein–Primakov transformation that finite Hund's coupling is responsible for the spin-wave softening in the ferromagnetic B-phase manganites. We obtain an analytical result for the spin-wave spectrum for $J_H \gg t$. In the limit of infinite Hund's coupling, the spectrum is the conventional nearest-neighbour Heisenberg ferromagnetic spin-wave. The $o(t/J_H)$-order correction is negative and thus accounts for the softening near the zone boundary.

(Some figures in this article are in colour only in the electronic version)

The observations of large magnetoresistance (LMR) in $Nd_{0.5}Pb_{0.5}MnO_3$, giant magnetoresistance (GMR) and colossal magnetoresistance (CMR) in manganites ($R_{1-x}A_xMnO_3$, R is a rare earth element and A a divalent alkaline-earth metal) a decade ago [1] have rekindled much interest in these materials which have been known for half a century [2]. Upon doping, the manganites undergo complicated transitions resulting in various magnetic, charge-ordering and orbital-ordering phases, showing the interplay between relevant spin, charge and orbital degrees of freedom. In particular, magnetism and electronic transport are clearly correlated. So it is widely believed that knowledge of the spin dynamics can provide important information on the underlying physics of CMR. Perring *et al* first measured the spin-waves in $La_{0.7}Pb_{0.3}MnO_3$ for a broad range of q [3]. The magnon spectrum is well defined at low temperatures and can be accounted for by the nearest-neighbour Heisenberg model. Subsequent measurements for $Pr_{0.63}Sr_{0.37}MnO_3$ and $Nd_{0.7}Sr_{0.3}MnO_3$ [4], $Nd_{0.7}Ba_{0.3}MnO_3$ [5] showed that the magnon spectrum deviates from the Heisenberg model and becomes softened near the zone boundary. So the behaviour seems a universal phenomenon of manganites.

As is well known, a number of interactions such as spin-orbital coupling, Hund's coupling, antiferromagnetic coupling between core spins, Coulomb interaction and dynamic Jahn–Teller

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effect coexist in manganites. These interactions are supposed to explain the existence of different phases of doped manganites. To explain the spin-wave softening, various mechanisms were proposed. The authors of [4] further showed that the experimental spectrum can be reproduced reasonably well by an extended Heisenberg model. Furukawa [6] argued that the softening seems to be explainable by a ferromagnetic Kondo lattice model with bandwidth narrower than the Hund's coupling. Solovyev *et al* [7] showed that the spin-wave behaviour near the zone boundary has a purely magnetic spin origin, and neither the lattice deformation nor the orbital ordering are required to account for the softening. Dai *et al* argued that the observed magnon softening and broadening are due to strong magnetoelastic interactions [8]. And this magnon–phonon coupling was later treated quantitatively in [9]. Using the ferromagnetic Kondo lattice model and composite operator method, Mancini *et al* obtained the softening spectrum [10]. Shannon *et al* constructed a theory of spin-wave excitations in the bilayer manganite $La_{1.2}Sr_{1.8}Mn_2O_7$ based on the simplest double-exchange model and partly explained the softening behaviour [11]. Krivenko *et al* showed that the scattering of spin excitations by low-lying orbital modes may cause the magnon softening [12].

In this paper, we show that in the hole-doped manganites, the softening behaviour might be of a purely electronic origin,i.e., a strong but finite Hund's coupling between the e_g electron and the core spin. Since in the hole-doped manganite there is less than one e_g electron per site on average and the $d_{x^2-y^2}$ orbital energy is significantly higher than that of $d_{3z^2-r^2}$ [13] due to Jahn–Teller splitting, a one-orbital description is a reasonable approximation. As in [14], we adopt the model Hamiltonian

$$H = t\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\sum_{\sigma} c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{j}\sigma} - J_{\mathrm{H}}\sum_{\mathbf{i}}\mathbf{s_i}\cdot\mathbf{S_i} + J_{\mathrm{AF}}\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\mathbf{S_i}\cdot\mathbf{S_j} - \mu\sum_{\mathbf{i}\sigma} c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{i}\sigma} + U\sum_{\mathbf{i}} n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow} \tag{1}$$

where t is the double exchange hopping, $\langle \mathbf{i},\mathbf{j}\rangle$ are nearest sites, μ is the chemical potential for the fermions, $c_{\mathbf{i}\sigma}$ represents the e_g electrons, J_{H} is the Hund's coupling between the e_g spin $\mathbf{s_i} = \frac{1}{2}c^{\dagger}_{\mathbf{i}}\boldsymbol{\sigma}c_{\mathbf{i}}$ and the core spin $\mathbf{S_i}$. J_{AF} is the antiferromagnetic interaction between the core spins, which is necessary to account for the G-phase parent ($x = 1$) manganites. The last term is the Hubbard Coulomb interaction. We use the Holstein–Primakov transformation for the core spins ($S = 3/2$); $S^{+}_{\mathbf{i}} = (2S - a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}})^{1/2}a_{\mathbf{i}}$, $S^{-}_{\mathbf{i}} = a^{\dagger}_{\mathbf{i}}(2S - a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}})^{1/2}$, $S^{z}_{\mathbf{i}} = S - a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}}$ and take the approximation $(2S - a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}})^{1/2} \simeq (2S - \langle a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}}\rangle)^{1/2}$. Homogeneity implies that $\langle a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}}\rangle = \langle a^{\dagger}a\rangle$. Because we consider the low-temperature case, we can drop the magnon quadratic term; hence the total Hamiltonian can be written as

$$\begin{aligned} H = {} & t\sum_{\langle \mathbf{i},\mathbf{j}\rangle}\sum_{\sigma} c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{j}\sigma} - \mu\sum_{\mathbf{i}\sigma} c^{\dagger}_{\mathbf{i}\sigma}c_{\mathbf{i}\sigma} + U\sum_{\mathbf{i}} n_{\mathbf{i}\uparrow}n_{\mathbf{i}\downarrow} - \tfrac{1}{2}J_{\mathrm{H}}A\sum_{\mathbf{i}}(s^{+}_{\mathbf{i}}a^{\dagger}_{\mathbf{i}} + s^{-}_{\mathbf{i}}a_{\mathbf{i}}) - J_{\mathrm{H}}S\sum_{\mathbf{i}} s^{z}_{\mathbf{i}} \\ & + A^2 J_{\mathrm{AF}}\sum_{\langle \mathbf{i},\mathbf{j}\rangle} a_{\mathbf{i}}a^{\dagger}_{\mathbf{j}} + J_{\mathrm{AF}}ZNS^2 - 2ZJ_{\mathrm{AF}}S\sum_{\mathbf{i}} a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}} + J_{\mathrm{H}}\sum_{\mathbf{i}} s^{z}_{\mathbf{i}}a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}}, \end{aligned} \tag{2}$$

where $A^2 = 2S - \langle a^{\dagger}a\rangle$, $Z = 6$ is the coordination number of the core spins. To use the composite operator method, we consider the doublet $B(\mathbf{i}) = (a_{\mathbf{i}}, \quad s^{+}_{\mathbf{i}})^{\mathrm{T}}$. The equation of motion for $B(\mathbf{i})$ is

$$\mathrm{i}\partial_t B(\mathbf{i}) = [B(\mathbf{i}), H] = \begin{pmatrix} -\frac{1}{2}J_{\mathrm{H}}As^{+}_{\mathbf{i}} + A^2 J_{\mathrm{AF}}\sum_{\mathbf{e}} a_{\mathbf{i}+\mathbf{e}} - 2ZJ_{\mathrm{AF}}Sa_{\mathbf{i}} + J_{\mathrm{H}}s^{z}_{\mathbf{i}}a_{\mathbf{i}} \\ t\sum_{\mathbf{e}}(c^{\dagger}_{\mathbf{i}\uparrow}c_{\mathbf{i}+\mathbf{e}\downarrow} - c^{\dagger}_{\mathbf{i}+\mathbf{e}\uparrow}c_{\mathbf{i}\downarrow}) - J_{\mathrm{H}}As^{z}_{\mathbf{i}}a_{\mathbf{i}} + J_{\mathrm{H}}Ss^{+}_{\mathbf{i}} - J_{\mathrm{H}}s^{+}_{\mathbf{i}}a^{\dagger}_{\mathbf{i}}a_{\mathbf{i}} \end{pmatrix}. \tag{3}$$

The composite operator method assumes that the right-hand side can be expressed as

$$[B(\mathbf{i}), H] = \sum_{\mathbf{j}} \varepsilon(\mathbf{i},\mathbf{j})B(\mathbf{j}) \tag{4}$$

with $\varepsilon(\mathbf{i},\mathbf{j})$ determined in the following way:

$$\varepsilon(\mathbf{i},\mathbf{j}) = \sum_{\mathbf{l}} m(\mathbf{i},\mathbf{l}) I^{-1}(\mathbf{l},\mathbf{j}) \tag{5}$$

where $I(\mathbf{i},\mathbf{j}) = \langle [B(\mathbf{i}), B^{\dagger}(\mathbf{j})] \rangle$, $m(\mathbf{i},\mathbf{j}) = \langle [\mathrm{i}\partial_t B(\mathbf{i}), B^{\dagger}(\mathbf{j})] \rangle$, and $\langle\ \rangle$ represents the expectation value. Thus $\varepsilon(\mathbf{i},\mathbf{j})$ contains some parameters to be determined self-consistently. This approach was proposed for the Hubbard model originally [14], and recent intensive studies [15] show credible agreement with Monte Carlo method. In our case (again due to homogeneity, $\langle s^z_{\mathbf{i}} \rangle = \langle s^z \rangle$)

$$\begin{aligned}
I(\mathbf{i},\mathbf{j}) &= \delta_{\mathbf{ij}} \cdot \mathrm{diag}(1, 2\langle s^z \rangle) \\
m_{11}(\mathbf{i},\mathbf{j}) &= \delta_{\mathbf{ij}}(J_{\mathrm{H}}\langle s^z \rangle - 2ZSJ_{\mathrm{AF}}) + A^2 J_{\mathrm{AF}} \sum_{\mathbf{e}} \delta_{\mathbf{j},\mathbf{i+e}} \\
m_{12}(\mathbf{i},\mathbf{j}) &= \delta_{\mathbf{ij}} J_{\mathrm{H}}(-A\langle s^z \rangle - \langle s^-_{\mathbf{i}} a_{\mathbf{i}} \rangle) \\
m_{22}(\mathbf{i},\mathbf{j}) &= -tp_1 \sum_{\mathbf{e}} (\delta_{\mathbf{ij}} - \delta_{\mathbf{j},\mathbf{i+e}}) + J_{\mathrm{H}} A \langle s^-_{\mathbf{i}} a_{\mathbf{i}} \rangle + 2J_{\mathrm{H}} S \langle s^z \rangle - 2J_{\mathrm{H}} \langle s^z_{\mathbf{i}} a^{\dagger}_{\mathbf{i}} a_{\mathbf{i}} \rangle
\end{aligned}$$

where $p_1 = \sum_{\sigma} \langle c^{\dagger}_{\mathbf{i}\sigma} c_{\mathbf{i}\sigma} \rangle$, $p_2 = \langle s^-_{\mathbf{i}} a_{\mathbf{i}} \rangle$, $p_3 = \langle s^z_{\mathbf{i}} a^{\dagger}_{\mathbf{i}} a_{\mathbf{i}} \rangle$. In the $\mathbf{k}$-space

$$\begin{aligned}
m_{11}(\mathbf{k}) &= (J_{\mathrm{H}}\langle s^z \rangle - 2ZSJ_{\mathrm{AF}}) + ZA^2 J_{\mathrm{AF}} \gamma_{\mathbf{k}} \\
m_{12}(\mathbf{k}) &= J_{\mathrm{H}}(-A\langle s^z \rangle - p_2) \\
m_{22}(\mathbf{k}) &= J_{\mathrm{H}} A p_2 - tZp_1(1-\gamma_{\mathbf{k}}) + 2J_{\mathrm{H}} S \langle s^z \rangle - 2J_{\mathrm{H}} p_3.
\end{aligned}$$

We assume that at $T = 0$ K, $\langle a^{\dagger} a \rangle = 0$, which satisfies self-consistency using the resulting retarded Green function and spectral theorem. Then the condition $\omega_{|\mathbf{k}=0} = 0$ requires that $p_3 = -\frac{1}{2} p_2 (A + \frac{p_2}{\langle s^z \rangle})$. So the ε-matrix is

$$\begin{aligned}
\varepsilon_{11}(\mathbf{k}) &= J_{\mathrm{H}}\langle s^z \rangle - 2ZSJ_{\mathrm{AF}}(1-\gamma_{\mathbf{k}}) \\
\varepsilon_{12}(\mathbf{k}) &= -\frac{J_{\mathrm{H}}}{2\langle s^z \rangle}(A\langle s^z \rangle + p_2) \\
\varepsilon_{21}(\mathbf{k}) &= -J_{\mathrm{H}}(A\langle s^z \rangle + p_2) \\
\varepsilon_{22}(\mathbf{k}) &= J_{\mathrm{H}} S - \frac{tZp_1}{2\langle s^z \rangle}(1-\gamma_{\mathbf{k}}) + J_{\mathrm{H}} A \frac{p_2}{\langle s^z \rangle} + J_{\mathrm{H}} \frac{p_2^2}{2\langle s^z \rangle^2}
\end{aligned}$$

and the Green function is

$$\begin{aligned}
D_{11}(\omega,\mathbf{k}) &= \frac{\omega - (J_{\mathrm{H}} S - \frac{tZp_1}{2\langle s^z \rangle}(1-\gamma_{\mathbf{k}}) + J_{\mathrm{H}} A \frac{p_2}{\langle s^z \rangle} + J_{\mathrm{H}} \frac{p_2^2}{2\langle s^z \rangle^2})}{(\omega - \omega_1(\mathbf{k}))(\omega - \omega_2(\mathbf{k}))} \\
D_{12}(\omega,\mathbf{k}) = D_{21}(\mathbf{k}) &= \frac{-J_{\mathrm{H}}(A\langle s^z \rangle + p_2)}{(\omega - \omega_1(\mathbf{k}))(\omega - \omega_2(\mathbf{k}))} \\
D_{22}(\omega,\mathbf{k}) &= \frac{2\langle s^z \rangle [\omega - (J_{\mathrm{H}}\langle s^z \rangle - 2ZSJ_{\mathrm{AF}}(1-\gamma_{\mathbf{k}}))]}{(\omega - \omega_1(\mathbf{k}))(\omega - \omega_2(\mathbf{k}))}
\end{aligned}$$

where $\omega_{1,2}(\mathbf{k})$ are acoustical and optical branches of the spin excitations. Using

$$p_2 = \frac{1}{N} \sum_{\mathbf{k}} \frac{\mathrm{i}}{2\pi} \int \mathrm{d}\omega \lim_{\eta \to 0} \frac{D_{12}(\omega + \mathrm{i}\eta, \mathbf{k}) - D_{12}(\omega - \mathrm{i}\eta, \mathbf{k})}{\mathrm{e}^{\beta\omega} - 1} \tag{6}$$

we have at $T = 0$ K, $p_2 = 0$; therefore, $p_3 = 0$. Accordingly, in this scheme, there are two parameters left: $\langle s^z \rangle$, p_1. The acoustical magnon spectrum can be expanded as a Taylor series which manifests the role of Hund's coupling

$$\omega_1(\mathbf{k}) = t \sum_{n=0}^{\infty} \left(\frac{t}{J_{\mathrm{H}}} \right)^n a_n (1-\gamma_{\mathbf{k}})^{n+1} \tag{7}$$

where, as usual, $\gamma_{\mathbf{k}} = Z^{-1}\sum_{\mathbf{e}} e^{i\mathbf{k}\cdot\mathbf{e}}$. The first few a_n are

$$a_0 = -\frac{3(4S^2\frac{J_{AF}}{t} + p_1)}{\langle s^z\rangle + S}$$

$$a_1 = -\frac{9S(4S\frac{J_{AF}}{t}\langle s^z\rangle - p_1)^2}{\langle s^z\rangle(\langle s^z\rangle + S)^3}$$

$$a_2 = \frac{27(4S\frac{J_{AF}}{t}\langle s^z\rangle - p_1)^3 S(S - \langle s^z\rangle)}{\langle s^z\rangle^2(S + \langle s^z\rangle)^5}$$

$$a_3 = -\frac{81(S^2 - 3S\langle s^z\rangle + \langle s^z\rangle^2)(4S\frac{J_{AF}}{t}\langle s^z\rangle - p_1)^4 S}{\langle s^z\rangle^3(S + \langle s^z\rangle)^7}.$$

In the small-k limit, $\omega \simeq Dk^2$, $D = -(4S^2\frac{J_{AF}}{t} + p_1)/(\langle s^z\rangle + S)$. Note that the hopping energy tp_1 is negative, and when it overcomes the AF term, the resulting magnon stiffness D is positive. Our numerical results show that this self-consistency is satisfied. Expression (7) suggests that the softening comes from the finite J_H. To fix the parameters $\langle s^z\rangle$, we use the spectral theorem and get $\langle s^z\rangle = \frac{1}{2}(1-x)$, where x is the dopant concentration. To fix p_1, we need the fermion sector. Using the notations in [10] for the fermion operator $\psi(\mathbf{i}) = (\xi_{\uparrow\mathbf{i}}, \eta_{\uparrow\mathbf{i}}, \xi_{\downarrow\mathbf{i}}, \eta_{\downarrow\mathbf{i}})^T$, where $\xi_\sigma = (1-n_{-\sigma})c_\sigma$, $\eta_\sigma = n_{-\sigma}c_\sigma$ are the Hubbard operators, we obtain the retarded Green function for ψ in the large-U limit at zero temperature.

$$G^R(\omega, \mathbf{k}) = \mathrm{diag}\left(\frac{1}{\omega - E_1(\mathbf{k})}, 0, \frac{x}{\omega - E_3(\mathbf{k})}, \frac{1-x}{\omega - E_4(\mathbf{k})}\right) \tag{8}$$

('diag' means diagonal matrix) with $E_1(\mathbf{k}) = -\mu + 6t\gamma_{\mathbf{k}} - \frac{1}{2}SJ_H$, $E_2(\mathbf{k}) = U - \mu + 6tu + 6tv\gamma_{\mathbf{k}}$, $E_3(\mathbf{k}) = [24tx\gamma_{\mathbf{k}} - 2\mu x + SJ_Hx + 12tp_\downarrow\gamma_{\mathbf{k}} - 12t\gamma_{\mathbf{k}} + 12t\Delta_\uparrow]/(2x)$, $E_4(\mathbf{k}) = [-2Ux + 2\mu x - SJ_Hx + 2U + 12tp_\downarrow\gamma_{\mathbf{k}} + SJ_H - 2\mu + 12t\Delta_\uparrow]/[2(1-x)]$, where Δ is related to the nearest-neighbour correlations of the Hubbard operators: $\Delta_\sigma = \langle\xi_\sigma(\mathbf{i}+\mathbf{e})\xi_\sigma^\dagger(\mathbf{i})\rangle - \langle\eta_\sigma(\mathbf{i}+\mathbf{e})\eta_\sigma^\dagger(\mathbf{i})\rangle$. In this scenario, E_1, E_3 are partially filled and E_2, E_4 are empty. The relevant parameters are μ, $\Delta_\uparrow$, $p_\downarrow$. We have three equations to fix them: $1 - x = 2 - C_{11}^F - C_{22}^F - C_{33}^F - C_{44}^F$, $\Delta_\uparrow = C_{11}^{F\gamma}$, $C_{11}^F = C_{33}^F$, where $C^F = \langle\psi(\mathbf{i})\psi^\dagger(\mathbf{i})\rangle$, $C^{F\gamma} = \langle\psi(\mathbf{i}+\mathbf{e})\psi^\dagger(\mathbf{i})\rangle$. We know that $C_{22}^F = 0$ and $C_{44}^F = 1 - x$. Thus $C_{11}^F = x = C_{33}^F$, so E_3 is empty, i.e., only E_1 is partially filled. Hence only μ is relevant to our problem and it can be fixed by $x = N^{-1}\sum_{\mathbf{k}}\theta(E_1(\mathbf{k}))$, where $\theta(x)$ is the usual step function. The hopping energy is

$$tp_1 = -tC_{11}^{F\gamma} = \frac{t}{N}\sum_{\mathbf{k}}\theta(-E_1(\mathbf{k}))\gamma_{\mathbf{k}} < 0. \tag{9}$$

The other two parameters $(\Delta_\uparrow, p_\downarrow)$ can also be determined by $t\Delta_\uparrow = -tp_1 > 0$, $24tx + 12t(p_\downarrow - 1) = -2\mu x + SJ_Hx + 12t\Delta_\uparrow$. Further analysis show that, for $J_H > 2.5t$, the whole scheme is self-consistent. Figure 1 shows the two relevant fermion bands for $x = 0.301$, $t = 1$, $J_H = 3.0$ (in units of t).

It is seen from the magnon spectrum (7) that we can estimate the two model parameters t and J_H from measured data. Figure 2 shows the comparison between our calculated result for the prescribed antiferromagnetic coupling $J_{AF} = 0.01$ and the measured result at $T = 10$ K for $Pr_{0.63}Sr_{0.37}MnO_3$ in [4]. The solid curve in the left panel is the fit to a nearest-neighbour Heisenberg model and gives a value at the zone boundary of about 34.2 meV. This corresponds to the uppermost curve in the right panel. The comparison gives the hopping energy $t \simeq 0.462$ eV. The circles are the measured data, and they give the value at the zone boundary of about 23 meV, corresponding to the point 0.05 in the right panel. This point corresponds to $J_H \simeq 3.2t \simeq 1.48$ eV. Note that the ratio J_H/t is very close to the values of interaction from a number of [16, 17]. It is worth noting that the nearest-neighbour Heisenberg

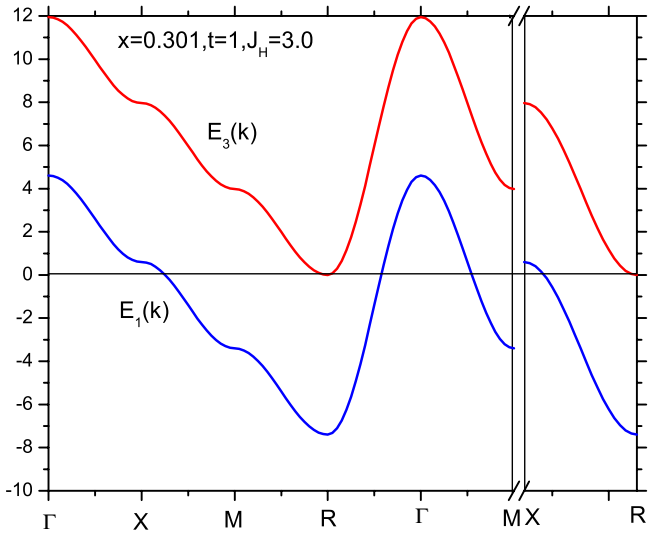

Figure 1. The bands for the Hubbard operators at $T = 0$ K.

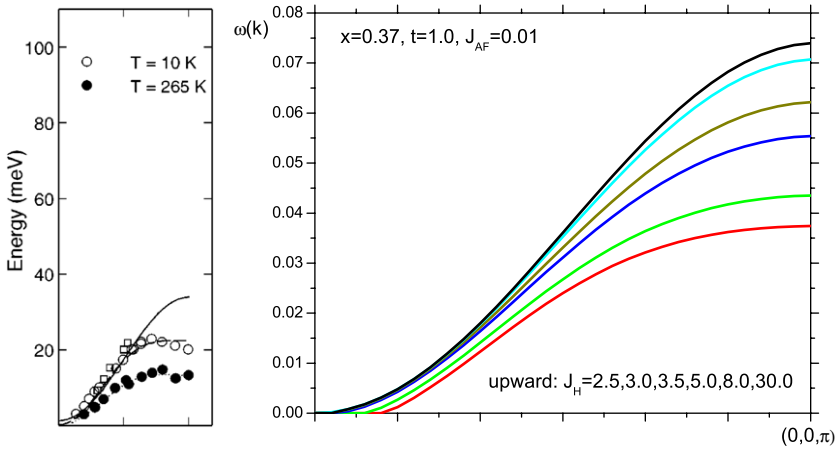

Figure 2. The spin-wave spectrum along ΓX of the B-phase. The left panel (taken from [4]) is the experimental result. The right panel is the calculated spin-wave spectrum (in units of t) at $T = 0$ K.

interaction alone cannot account for the Curie temperature. The fitting curve in the left panel corresponds to the nearest-neighbour Heisenberg spectrum $\omega(\mathbf{k}) \simeq 51.3(1 - \gamma_{\mathbf{k}})$ meV. In the mean field theory, the Curie temperature T_c corresponding to the spectrum $\omega_{\mathbf{k}} = 2ZS^*J(1-\gamma_{\mathbf{k}})$ is $k_B T_c = \frac{2}{3}JZS^*(S^* + 1)$ (here $S^* = S + \frac{1}{2}(1 - x)$ is the effective spin). This gives $T_C^{MF} \simeq 500$ K. Taking into account that in three dimensions for a simple cubic lattice, the real Curie temperature T_C and T_C^{MF} have the relation [18] $T_C = 0.75T_C^{MF}$, we get $T_c \simeq 375$ K, higher than the real value of 315 K. To conclude this paper, we present some discussions and comments. In the derivation of the series expression of the magnon spectrum, we have used the approximation $(2S - a_{\mathbf{i}}^\dagger a_{\mathbf{i}})^{1/2} \simeq (2S - \langle a_{\mathbf{i}}^\dagger a_{\mathbf{i}}\rangle)^{1/2}$ in the Holstein–Primakov transformation. This can be satisfied at very low temperatures. Further, the quartic term $J_{AF}\sum_{\langle \mathbf{i},\mathbf{j}\rangle} a_{\mathbf{i}}^\dagger a_{\mathbf{i}} a_{\mathbf{j}}^\dagger a_{\mathbf{j}}$

is neglected because J_{AF} is very small and the magnon fluctuation at zero temperature is negligible. The series expression (7) of the acoustic magnon dispersion shows alternating behaviour; convergence is guaranteed when $J_H S > 3$. The model parameters t, J_H, J_{AF} and the hopping energy p_1 can be estimated by fitting experimental data. There is a simple physical picture for the deviation of the magnon spectrum from that of the Heisenberg model. The interaction between core spins is induced by the hopping of e_g-electrons and the dominant term is linear in t. If the Hund's coupling J_H is infinite, only the dominant term plays the role. The e_g electron and core spin must add up to a total spin-2 to minimize the energy in the B-phase. So the actual background for spin excitation is just that in the simple Heisenberg model. But for finite J_H, high orders of the mediated interaction between core spins make some difference. Our result (7) agrees with the conclusions from the random phase approximation [19], which provides an integral equation for the dispersion relation. The strength of the induced ferromagnetic interaction is determined by the hopping energy of the conduction fermions. For the approach presented to be self-consistent, the spin-wave stiffness must be positive. The ferromagnetic order becomes unstable at a certain filling when the stiffness vanishes. However, zone boundary spin-wave softening can be explained by the spin dynamics in the Kondo lattice model, as shown in this paper. The origin of the behaviour is still an issue of debate. Based on the observed proximity of phonon dispersion and magnon dispersion and the anisotropic spin-wave broadening, Dai *et al* [8] argue that strong magnon–phonon coupling is needed for a complete understanding of the low-temperature spin dynamics of manganites. Quite recently, Endoh *et al* concluded [20] that the ferromagnetic magnons in $Sm_{0.55}Sr_{0.45}MnO_3$ are of orbital nature since the magnon dispersion shows anisotropy which is mainly determined by the short-range correlation of the e_g orbitals. They explained that the anisotropic magnon dispersion is attributed to long-range magnetic interactions based on fitting the data to a Heisenberg model with long-range interactions. We believe that if orbital degrees of freedom are taken into account in our model, the resulting magnon spectrum will be anisotropic, since orbital degrees of freedom bring anisotropy into the system. Finally, we remark that as manganites are very complex systems, there might be multiple mechanisms contributing to a single phenomenon. The analysis provided in this paper shows that Hund coupling might be of primary importance.

Acknowledgments

SSF is grateful to Professor P Schlottmann, Professor Mancini, and Dr Avella for their helpful discussions. This work is supported in part by the Army High Performance Computing Research Center (AHPCRC) under the auspices of the Department of the Army, Army Research Laboratory (ARL) under Cooperative Agreement number DAAD19-01-2-0014.

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